IS $(xe^{xy}-2y)\,dy+(ye^{xy}+2x)\,dx=0$ EXACT?

IF SO, SOLVE IT

$$\frac{\partial}{\partial x}(xe^{xy}-2y)=1\cdot e^{xy}+x\cdot ye^{xy}=e^{xy}(1+xy)$$
$$\frac{\partial}{\partial y}(ye^{xy}+2x)=1\cdot e^{xy}+y\cdot xe^{xy}=e^{xy}(1+xy)$$

$=$

EQ'N IS EXACT

SO, $F_y=xe^{xy}-2y$

$$\frac{\partial F}{\partial y}=xe^{xy}-2y$$
$$F=\int(xe^{xy}-2y)\,dy$$
$$=e^{xy}-y^2+C(x)$$
$$=e^{xy}+C(x)-y^2$$

$F_x=ye^{xy}+2x$

$$\frac{\partial F}{\partial x}=ye^{xy}+2x$$
$$F=\int(ye^{xy}+2x)\,dx$$
$$=e^{xy}+x^2+K(y)$$

$C(x)=x^2 \quad K(y)=-y^2$

$$F=e^{xy}+x^2-y^2$$

$e^{xy}+x^2-y^2=C$ IS THE SOL'N OF THE DE

OR (NEXT PAGE)

OR

$$F_y = xe^{xy} - 2y$$

$$\frac{\partial F}{\partial y} = xe^{xy} - 2y$$

$$F = \int (xe^{xy} - 2y)\,dy$$

$$F = e^{xy} - y^2 + C(x)$$

$$F_x = \cancel{ye^{xy}} + C'(x) = \underbrace{\cancel{ye^{xy}} + 2x}_{\text{FROM DE}}$$

$$C'(x) = 2x \leftarrow$$ MANDATORY CHECKPOINT: CONTAINS ONLY X

$$C(x) = \int 2x\,dx$$

$$C(x) = x^2$$

$$F = e^{xy} - y^2 + x^2$$

SOL'N IS $e^{xy} - y^2 + x^2 = C$

OR

$F_x = ye^{xy} + 2x$

$\frac{\partial F}{\partial x} = ye^{xy} + 2x$

$F = \int (ye^{xy} + 2x)\,dx$

$F = e^{xy} + x^2 + K(y)$

$F_y = \cancel{xe^{xy}} + K'(y) = \cancel{xe^{xy}} - 2y$

$K'(y) = -2y$ ← MANDATORY CHECKPOINT ONLY CONTAINS y

$K(y) = \int -2y\,dy$

$K(y) = -y^2$

$F = e^{xy} + x^2 - y^2$

SOL'N IS $e^{xy} + x^2 - y^2 = C$

SOLVE $\dfrac{dy}{dx} = -\dfrac{y^2}{2xy+\cos y}$ $\xrightarrow{\text{OR}}$ $\dfrac{dx}{dy} = -\dfrac{2xy+\cos y}{y^2}$

$$\frac{dx}{dy} = -\frac{2}{y}x - \frac{\cos y}{y^2}$$

$$\frac{dx}{dy} + \frac{2}{y}x = -\frac{\cos y}{y^2}$$

LINEAR $\rightarrow$ $\mu = e^{\int \frac{2}{y}\,dy} = y^2$

$$(2xy+\cos y)\,dy = -y^2\,dx$$

$$y^2\,dx + (2xy+\cos y)\,dy = 0$$

$$\frac{\partial y^2}{\partial y} = 2y = \frac{\partial(2xy+\cos y)}{\partial x} \longrightarrow \text{EQ'N IS EXACT}$$

$F_x = y^2$ $\quad F = \int y^2\,dx = xy^2 + C(y)$

$$F_y = \cancel{2xy} + C'(y) = \cancel{2xy} + \cos y$$

$$C'(y) = \cos y \longleftarrow$$ MANDATORY CHECKPOINT: CONTAINS ONLY y

$$C(y) = \int \cos y\,dy = \sin y$$

$$F = xy^2 + \sin y$$

$xy^2 + \sin y = C$ IS THE SOL'N

$$(3xy+2y^2)dx + (2x^2+3xy)dy = 0$$

$$\frac{\partial(3xy+2y^2)}{\partial y} = 3x+4y \neq \frac{\partial(2x^2+3xy)}{\partial x} = 4x+3y$$

NOT EXACT

GUESS: $\mu(x,y) = x^a y^b \quad a,b \in \mathbb{R}$

$$x^a y^b \left[(3xy+2y^2)\,dx + (2x^2+3xy)\,dy \right] = [0]x^a y^b$$

$$\underbrace{(3x^{a+1}y^{b+1} + 2x^a y^{b+2})}_{M} dx + \underbrace{(2x^{a+2}y^b + 3x^{a+1}y^{b+1})}_{N} dy = 0$$

WANT THIS TO BE EXACT

$$\frac{\partial M}{\partial y} = 3(b+1)x^{a+1}y^b + 2(b+2)x^a y^{b+1}$$

$$\frac{\partial N}{\partial x} = 2(a+2)x^{a+1}y^b + 3(a+1)x^a y^{b+1}$$

NEED THESE TO BE EQUAL

$3(b+1) = 2(a+2)$ AND $2(b+2) = 3(a+1)$

$3b+3 = 2a+4 \qquad 2b+4 = 3a+3$

$0 = 2a-3b+1 \qquad 0 = 3a-2b-1$

$2(2a-3b)=(-1)2$
$-3(3a-2b)=(1)-3$

$$\begin{aligned} 4a-6b&=-2\\ -9a+6b&=-3\\ \hline -5a\quad&=-5 \longrightarrow a=1 \end{aligned}$$

$3-2b=1$
$-2b=-2$
$b=1$

$\mu=x^a y^b = xy$

$xy\left[(3xy+2y^2)\,dx+(2x^2+3xy)\,dy\right]=0$

$\underbrace{(3x^2y^2+2xy^3)}_{M}dx+\underbrace{(2x^3y+3x^2y^2)}_{N}dy=0$

MANDATORY CHECKPOINT: EQ'N IS NOW EXACT

$M_y=6x^2y+6xy^2=N_x=6x^2y+6xy^2$

$F=\int M\,dx=\int(3x^2y^2+2xy^3)\,dx$

$F=x^3y^2+x^2y^3+C(y)$

$F_y=\cancel{2x^3y}+\cancel{3x^2y^2}+C'(y)=\cancel{2x^3y}+\cancel{3x^2y^2}$

$C'(y)=0$ MANDATORY CHECKPOINT: ONLY CONTAINS y

$C(y)=0$

$x^3y^2+x^2y^3=C$ IS SOL'N